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Absolute instability of a liquid jet in a gas

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The effect of the ambient gas density on the onset of absolute instability in a viscous liquid jet is examined. The critical Weber number, above which the instability is convective and below which the instability is absolute, is determined as a function of Reynolds number and the density ratio of gas to liquid. It is shown that the gas density has the effect of raising the critical Weber number. It also raises the cutoff wavenumber below which disturbances are spatially amplified and above which they are damped.

I. INTRODUCTION

The capillary instability of an infinitely long jet with respect to temporally growing disturbances was analyzed by Rayleigh.¹ Keller² *et al.* examined the capillary instability of a semi-infinite jet with respect to spatially growing disturbances. They found that the temporal and spatial disturbances are analytically related if the Weber number is sufficiently large. For sufficiently small Weber numbers, Leib and Goldstein³ found that the state of convective instability obtained by Keller *et al.* actually cannot be reached by a given initial disturbance in the sense of Briggs⁴ and Bers.⁵ Leib and Goldstein demonstrated, for the first time, the existence of the absolute instability in an inviscid jet. Recently, they⁶ determined from Chandrasekhar's⁷ dispersion equation the critical Weber number below which a viscous jet is absolutely unstable as a function of Reynolds number. They also found that the cutoff wavenumber, above which the disturbance is spatially damped, is one independent of the Weber number.

Here, we examine the effect of the ambient gas density on the absolute instability discovered by Leib and Goldstein. It is shown that the gas density has the effect of enlarging the domain of absolute instability in the Reynolds-Weber number space. Moreover, it raises the cutoff wavenumber below which the jet is convectively unstable, and also raises the amplification rate of the spatially growing disturbances when the absolute instability is absent.

II. FORMULATION

Consider a circular cylindrical jet of an incompressible viscous Newtonian liquid issued from a nozzle into an unbounded inviscid gas. The governing dynamic equations of motion in the liquid and the gas phases are, respectively, the Navier-Stokes and Euler equations. The boundary conditions are the vanishing of the net force per unit area of the interface, and the equality of radial fluid velocity in each phase with the total time rate of change of the interfacial position. A uniform velocity distribution U in a circular jet of radius r_0 in a quiescent gas is an exact solution to this set of differential equations in the absence of gravity. This exact mathematical solution representing a possible basic state is physically unstable. The stability analysis of this basic state with respect to any Fourier component of the disturbances

of the form $C_0 \exp(\omega\tau +iky)$ led to the characteristic equation (1) when viewed in a reference frame with its origin fixed at the nozzle exit.^{8,9}

$$(\omega - ik)^2 + \frac{2k^2}{R} \left(\frac{I_1'(k)}{I_0(k)} - \frac{2k\lambda}{\lambda^2 + k^2} \frac{I_1(k)I_1'(\lambda)}{I_0(k)I_0(\lambda)} \right) \\ \times (\omega - ik) + \omega^2 \frac{\rho_2}{\rho_1} \frac{\lambda^2 - k^2}{\lambda^2 + k^2} \frac{K_0(k)I_1(k)}{K_1(k)I_0(k)} \\ - \frac{1}{\beta} k(1 - k^2) \frac{\lambda^2 - k^2}{\lambda^2 + k^2} \frac{I_1(k)}{I_0(k)} = 0, \quad (1)$$

where C_0 is the wave amplitude, ω is the complex wave frequency (the real part of which gives the exponential temporal growth rate, and imaginary part of which gives the wave frequency of disturbances), k is the complex wavenumber [the real part of which is equal to $2\pi r_0/(\text{wavelength})$ and the imaginary part of which gives the spatial amplification rate], τ is time normalized with r_0/U , y is the axial distance measured in the unit of r_0 in the opposite direction of the jet flow, I and K are, respectively, the modified Bessel functions of the first and second kind, their subscripts denote the order of the functions, and λ is defined by

$$\lambda^2 = k^2 + R(\omega - ik).$$

The three independent flow parameters in (1) are Reynolds number R , Weber number β , and the density ratio Q defined, respectively, by

$$R = Ur_0/\nu, \quad \beta = \rho_1 U^2 r_0/\sigma, \quad Q = \rho_2/\rho_1,$$

where ν is kinematic viscosity, ρ is density, σ is surface tension, and the subscripts 1 and 2 denote, respectively, the liquid and gas phases. Note that the above nondimensionalization of time implies that the dimensional frequency is given by $\omega U/r_0$.

III. RESULTS

Figure 1 shows the loci of the two characteristic roots obtained when a given set of flow parameters as the temporal growth rate ω_r varies for a number of values of the wave frequency ω_i . As the ω_r is reduced from a finite positive value to zero, these loci approach toward two different branches of amplification curves $\omega_r = 0$ for the spatially growing disturbances. Note that $k_i > 0$ gives the spatial instability because the y axis is chosen to be negative in the flow direction. It can be seen that two of the loci with

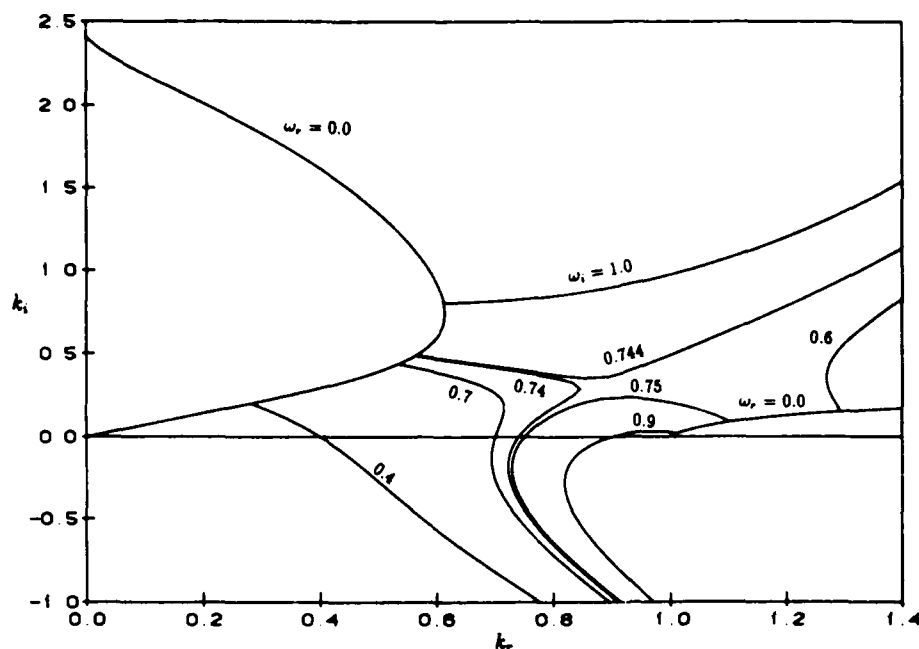


FIG. 1. Absolute instability: $Q = 0.0013$, $\beta = 1$, $R = 20$.

$0.74 < \omega_i < 0.75$ will intersect in the upper half-plane at a saddle point.⁵ A similar situation has already been shown by Leib and Goldstein³ for the case of negligible gas density. As the Weber number is increased, with the values of Q and R given in Fig. 1 being fixed, this saddle point moves closer to the pure spatial amplification curve for disturbances with a "group velocity" in the direction of jet flow. Note that the amplification curve, which does not originate from $k_r = 0$, is for the disturbances with upstream propagating "group velocity." Following Leib and Goldstein, the minimum value of β for which the saddle point of Eq. (1) lies in the upper half-plane is defined as the critical Weber number. Figure 2

gives the dependence of the critical Weber number on R for three values of Q . Here, $Q = 0.0013$ corresponds to the air to water density ratio in one atmospheric pressure in room temperature. The curve $Q = 0$ was obtained by Leib and Goldstein. In the parameter range below the curves, the jet is absolutely unstable. The dashed line indicates the inviscid limit obtained by Leib and Goldstein for the case of $Q = 0$. It is seen that the gas density has the effect of enlarging the domain of absolute instability for the β - R plane.

Above these curves, the jet is convectively unstable. Figures 3 and 4 give a few typical amplification curves for convectively unstable disturbances. Leib and Goldstein showed

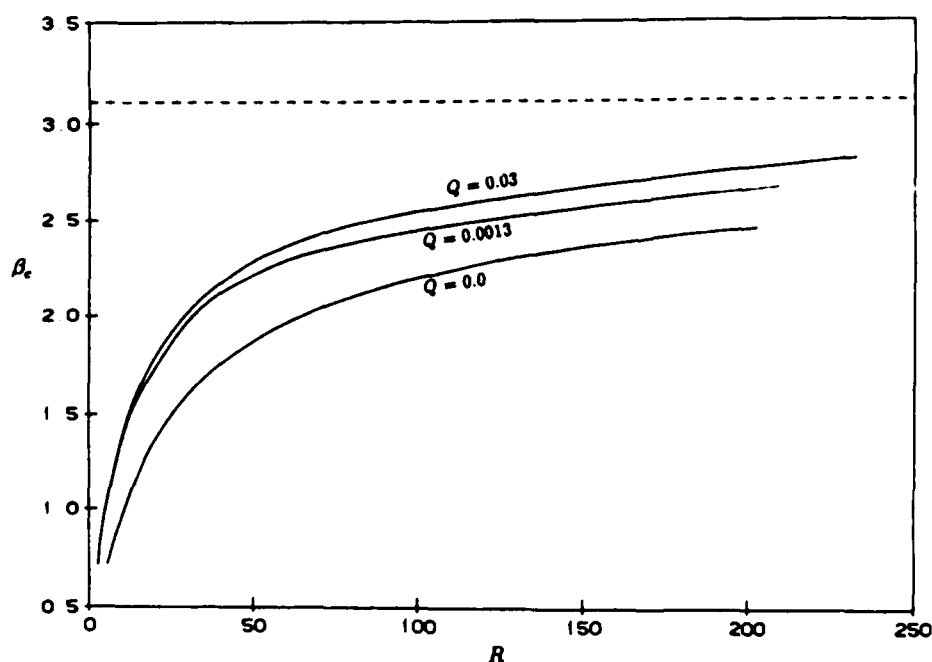


FIG. 2. The critical Weber number versus Reynolds number.

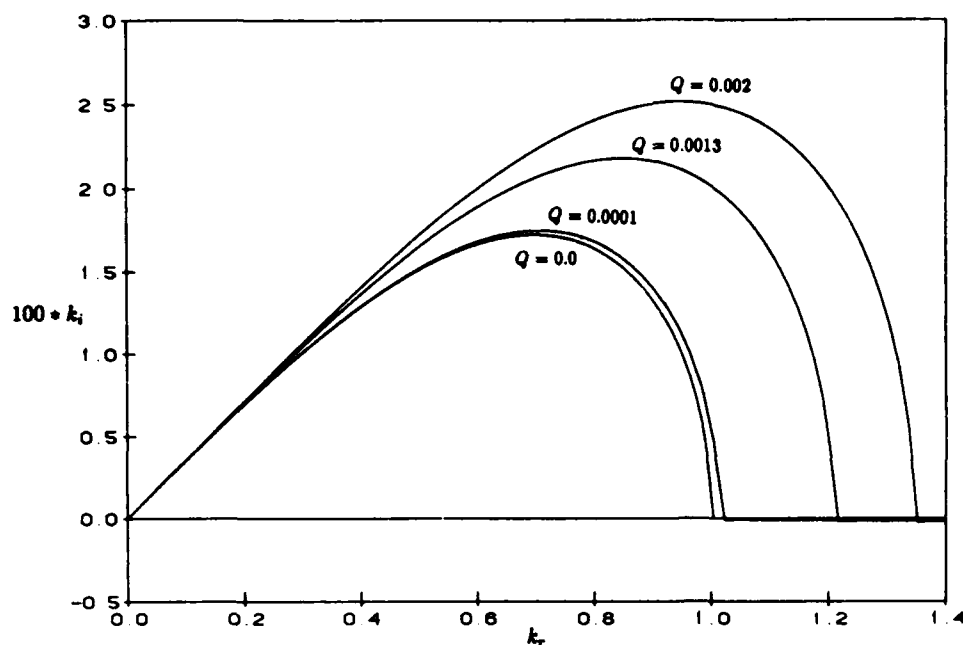


FIG. 3. The growth rate for convectively unstable disturbances: $R = 20\,000$, $\beta = 400$.

that the cutoff wavenumber, above which the disturbance is damped, is one independent of β for the case of $Q = 0$. It is seen from Figs. 3 and 4 that the gas density tends to raise the cutoff wavenumber very significantly. This has a practical significance of producing smaller droplets by use of larger ambient gas density.⁹

It is very hard to resist the temptation of speculating that the absolute instability may correspond to the dripping mode of the jet instability. When a saddle point of the eigenvalue exists at a certain frequency and there exist both upstream and downstream propagating convectively unstable branches, a disturbance with this certain frequency may be alternatively attracted toward these two branches as time

evolves. The simultaneous spatial and temporal growth alternating in the upstream and downstream directions with a regular frequency casually observed in a dripping jet seems to correspond to the mathematical picture of absolute instability. This speculation is even more tempting when one realizes that the absolute instability is predicted only for a small Weber number, which is the ratio of the inertial force to the surface tension force. However, the argument remains speculative, since the dripping phenomenon involves a highly nonlinear finite amplitude of disturbances that cannot be accounted for in the present linear theory. However, Leib and Goldstein³ suggested that the absolutely unstable disturbance may grow nonlinearly by the mechanism of direct res-

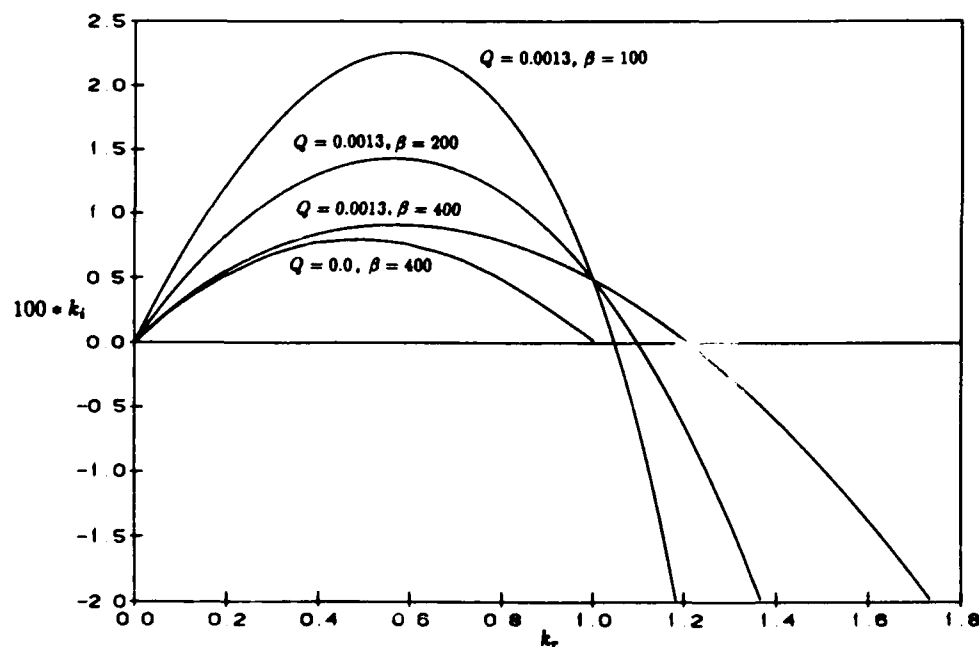


FIG. 4. The growth rate for convectively unstable disturbances, $R = 34.5$.

onance advanced by Akylas and Benney.¹⁰ The physical significance of the absolute instability in the context of nonlinear theories remains to be explored.

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